C 和の記号Σの性質

数列$\{a_n\}$，$\{b_n\}$および定数 p について，次の式が成り立つ。

1. $\displaystyle\sum_{k=1}^{n}(a_k+b_k)=\sum_{k=1}^{n}a_k+\sum_{k=1}^{n}b_k$
2. $\displaystyle\sum_{k=1}^{n}pa_k=p\sum_{k=1}^{n}a_k$

(証明)

1. $\displaystyle\sum_{k=1}^{n}(a_k+b_k)=(a_1+b_1)+(a_2+b_2)+(a_3+b_3)+\cdots+(a_n+b_n)$

$=(a_1+a_2+a_3+\cdots+a_n)+(b_1+b_2+b_3+\cdots+b_n)$

$\displaystyle=\sum_{k=1}^{n}a_k+\sum_{k=1}^{n}b_k$

2. $\displaystyle\sum_{k=1}^{n}pa_k=pa_1+pa_2+pa_3+\cdots+pa_n$

$=p(a_1+a_2+a_3+\cdots+a_n)$

$\displaystyle=p\sum_{k=1}^{n}a_k$

〈例19〉 次の和を求めよ。

(1) $\displaystyle\sum_{k=1}^{n}(2k+3)=\sum_{k=1}^{n}2k+\sum_{k=1}^{n}3=2\sum_{k=1}^{n}k+\sum_{k=1}^{n}3$

$\displaystyle=2\times\frac{n(n+1)}{2}+3n=n^2+4n$

(2) $\displaystyle\sum_{k=1}^{n}(2k-1)=2\sum_{k=1}^{n}k-\sum_{k=1}^{n}1=2\times\frac{n(n+1)}{2}-n=n^2$

(3) $\displaystyle\sum_{k=1}^{n}(2k^2+3k-2)=\sum_{k=1}^{n}2k^2+\sum_{k=1}^{n}3k-\sum_{k=1}^{n}2$

$\displaystyle=2\times\frac{n(n+1)(2n+1)}{6}+3\times\frac{n(n+1)}{2}-2n$

$\displaystyle=\frac{4n^3+6n^2+2n+9n^2+9n-12n}{6}=\frac{4n^3+15n^2-n}{6}=\frac{n(4n^2+15n-1)}{6}$

(4) $\sum_{k=1}^{n-1}(4k+1)=4\sum_{k=1}^{n-1}k+\sum_{k=1}^{n-1}1$

$=4\times\frac{(n-1)n}{2}+(n-1)=2n^2-n-1$

演習６　次の和を求めよ。

(1) $\sum_{k=1}^{n}k(k+1)=\sum_{k=1}^{n}(k^2+k)=\sum_{k=1}^{n}k^2+\sum_{k=1}^{n}k$

$=\frac{n(n+1)(2n+1)}{6}+\frac{n(n+1)}{2}=n(n+1)\left(\frac{2n+1}{6}+\frac{1}{2}\right)$

$=n(n+1)\left(\frac{2n+1+3}{6}\right)$

$=n(n+1)\left(\frac{2n+4}{6}\right)$

$=\frac{n(n+1)(n+2)}{3}$

共通因数 n(n+1)
でくくる

(2) $1\cdot3+3\cdot5+5\cdot7+7\cdot9+\cdots\cdots+$(第n項)

$=\sum_{k=1}^{n}(2k-1)(2k+1)=\sum_{k=1}^{n}(4k^2-1)$

$=4\times\frac{n(n+1)(2n+1)}{6}-n$

$=2\times\frac{2n^3+3n^2+n}{3}-n$

$=\frac{4n^3+6n^2-n}{3}$

$=\frac{n(4n^2+6n-1)}{3}$

$a_1=1\cdot3$
$a_2=3\cdot5$
$a_3=5\cdot7$
$a_4=7\cdot9$
より、
$a_k=(2k-1)(2k+1)$